

# Hyperkahler quotients, singularities, and quivers: January 30 - February 3, 2023.

Events for:  
Monday, January 30th - Friday, February 3rd

## Monday, January 30th

9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Eyal Markman - SCGP 102**

**Speaker:** Eyal Markman

**Title:** Rational Hodge isometries of hyper-Kahler varieties of  $K3[n]$  and generalized Kummer type are algebraic

**Abstract:** Let  $X$  and  $Y$  be projective hyper-Kahler manifolds deformation equivalent to the Hilbert scheme of  $n$  points on a  $K3$  surface. Let  $f$  be a Hodge isometry of the second rational cohomologies of  $X$  and  $Y$  with respect to the Beauville-Bogomolov-Fujiki pairings. We prove that  $f$  is induced by an algebraic correspondence. We furthermore lift  $f$  to an algebraic correspondence  $F$  between their total rational cohomologies, which is a Hodge isometry with respect to the Mukai pairings, and which preserves the gradings up to sign. We will also discuss the analogous result for hyperkahler manifolds of generalized Kummer deformation type.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Nicolas Addington - SCGP 102**

**Speaker:** Nicolas Addington

**Title:** A failed attempt at irrationality via algebraic K-theory

**Abstract:** The Quillen-Lichtenbaum conjecture, proved by Voevodsky, states that for smooth complex  $n$ -folds, the map from algebraic to topological K-theory with finite coefficients is an isomorphism in degree  $n-1$  and higher, and injective in degree  $n-2$ . From this and some blow-up formulas one can construct a birational invariant. Surprisingly, it is somewhat computable. But sadly it vanishes for all cubic fourfolds; the proof repackages substantial cycle-theoretic results of Voisin, M. Shen, and others. Or to say it in a positive way, Kuznetsov's  $K3$  category behaves from this perspective like the derived category of an honest surface. This is work in progress with Elden Elmanto.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop Speaker: Benjamin Bakker - SCGP 102**

**Speaker:** Benjamin Bakker

**Title:** The Matsushita alternative

**Abstract:** In this talk I'll explain how to prove a conjecture of Matsushita that Lagrangian fibrations of irreducible compact hyperkahler manifolds are either isotrivial or vary maximally in moduli. I will also deduce some results about the density of torsion points in sections and discuss some motivating applications to Chow theory due to Voisin.

4:00pm **Workshop Speaker: Anindya Dey - SCGP 102**

**Speaker:** Anindya Dey

**Title:** Terminal singularities, Hyperkahler Quotients and Quiver N-ality

**Abstract:** In the program of geometric engineering, a very large class of non-Lagrangian 4d  $\mathcal{N}=2$  SCFTs are obtained by compactifying Type IIB String Theory on Calabi-Yau three-folds which are singular hypersurfaces in four complex dimensions. Generically, the Higgs branches of such SCFTs are difficult to determine because of non-trivial quantum corrections. However, an appropriate circle reduction of a 4d SCFT may lead to a 3d SCFT with a quiver description, and the resultant quiver then gives a realization of the 4d Higgs branch as a hyperkahler quotient. In this talk, I will present an explicit construction to show that the Higgs branch for a large subclass of these 4d SCFTs admits *more than one* hyperkahler quotient realization. A recently discovered N-ality (i.e. equivalence of N distinct theories) involving three dimensional quiver gauge theories plays a crucial role in the story. I will discuss how the property of 3d N-ality is encoded in the geometry of the CY three-fold used to engineer the 4d SCFT.

**Tuesday, January 31st**

10:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

11:00am **Workshop Speaker: Daniel Greb - SCGP 102**

**Speaker:** Daniel Greb

**Title:** Cycles in the K3 period domain and moduli of complex hyperkähler metrics

**Abstract:** Hyperkähler metrics on K3 surfaces give rise to rational curves of degree 2 in the K3 period domain, so-called "twistor lines". While these are used in the proofs of many deep results, their existence also implies that the group of isometries of the K3 lattice does not act properly discontinuously on the period domain, preventing a moduli space of unpolarised complex K3 surfaces to exist. I will report on work in progress with Martin Schwald (Essen), in which we study the cycle space of the K3 period domain. This space parametrises twistor lines as part of its real locus, but also all their degenerations and complex deformations as submanifolds of the period domain. I will explain how many foundational problems regarding the moduli theory of K3s disappear when passing to the cycle space and also indicate how the original version of Penrose's Twistor Theory (the "nonlinear graviton" construction) can be used to understand what kind of geometric structure a small complex deformation of an honest twistor line corresponds to.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop Speaker: Ekaterina Amerik - SCGP 102**

**Speaker:** Ekaterina Amerik

**Title:** Parabolic automorphisms of hyperkähler manifolds

**Abstract:** As observed by Serge Cantat, an automorphism of a K3 surface which is a fiberwise translation in an elliptic pencil acts with dense orbits in the fibers; as a consequence, if a K3 surface has two such automorphisms (with distinct pencils), then the group they generate acts ergodically. We show that the same holds for arbitrary hyperkähler manifolds, as an application of Deligne semisimplicity theorem. This is a joint work with Misha Verbitsky.

4:00pm **Workshop Speaker: Grégoire Menet - SCGP 102**

**Speaker:** Gregoire Menet

**Title:** Some open problems in order to construct compact hyperkähler orbifolds

**Abstract:** The hyperkähler orbifolds can be seen as one of the simplest generalization of the hyperkähler manifolds in the singular setting. In particular, such orbifolds can be constructed as terminalizations of some symplectic quotients. The objective of this talk is to provide the "state of the art" of these constructions.

<b>Wednesday, February 1st</b>
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9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Amihay Hanany - SCGP 102**

**Speaker:** Aminhay Hanany

**Title:** Symplectic singularities and Coulomb branches

**Abstract:** TBA

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Szilard Szabo - SCGP 102**

**Speaker:** Szilard Szabo

**Title:** Hitchin WKB-problem and  $P = W$  conjecture in lowest degree for rank 2 over the 5-punctured sphere

**Abstract:** We use abelianization of Higgs bundles away from the ramification divisor and fiducial solutions to analyze the large-scale behavior of Fenchel–Nielsen co-ordinates on the moduli space of rank 2 Higgs bundles on the Riemann sphere with five punctures. We solve the related Hitchin WKB problem and prove the lowest degree weighted pieces of the  $P = W$  conjecture in this case.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop Speaker: Chiara Camere - SCGP 102**

**Speaker:** Chiara Camere

**Title:** Prym fibrations as irreducible symplectic varieties

**Abstract:** In this talk, I will first recall the construction of Lagrangian fibrations by Prym varieties starting from a K3 surface with a non-symplectic involution. Then I will discuss a criterion to ensure that the normalization of such a fibration is an irreducible symplectic variety. This is joint work in progress with E. Brakkee, A. Grossi, L. Pertusi, G. Saccà and A. Viktorova.

4:00pm **Workshop Speaker: Lie Fu - SCGP 102**

**Speaker:** Lie Fu

**Title:** Derived equivalent K3 surfaces and their motives as algebra objects.

**Abstract:** We show that two (twisted) derived K3 surfaces have isomorphic rational Chow motives as Frobenius algebra objects. Combined with Huybrechts' result, we get a motivic characterization for two K3 surfaces to be isogenous. We raise some questions for higher-dimensional hyper-Kähler varieties. In the non-commutative direction, we prove an analogous relation between the motives of two cubic fourfolds with equivalent Kuznetsov component. The talk is based on a series of joint work with Charles Vial.

**Thursday, February 2nd**

9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Michal Kapustka - SCGP 102**

**Speaker:** Michal Kapustka

**Title:** A locally complete family of projective IHS orbifolds of Nikulin type

**Abstract:** Nikulin orbifolds are irreducible holomorphic symplectic orbifolds which are partial resolutions of quotients of IHS manifolds of  $K3^n$  type. Their deformations are called orbifolds of Nikulin type. I will describe the first known locally complete family of projective irreducible holomorphic symplectic orbifolds of dimension 4 which are of Nikulin type. It is a family of IHS orbifolds that appear as double covers of special complete intersections  $(3,4)$  in  $\mathbb{P}^6$ . This is joint work with Ch. Camere, A. Garbagnati and G. Kapustka.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Gianluca Pacienza - SCGP 102**

**Speaker:** Gianluca Pacienza

**Title:** On the cone conjecture for Enriques manifolds.

**Abstract:** Enriques manifolds are non simply connected manifolds whose universal cover is irreducible holomorphic symplectic, and as such they are natural generalizations of Enriques surfaces. In this talk I will report about a joint work in progress with Alessandra Sarti in which we study the Morrison-Kawamata cone conjecture for such manifolds using the analogous result (established by Amerik-Verbitsky) for their universal cover.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch

2:30pm **Workshop Speaker: Alessandra Sarti - SCGP 102**

**Speaker:** Alessandra Sarti

**Title:** Recent progress on the study of automorphisms of K3 surfaces

**Abstract:** I will talk about new results on automorphisms of order a multiple of seven acting purely non-symplectically on K3 surfaces. It turns out that the possible orders are 7, 14, 21, 28 and 42. These are some of the last few cases left out in view of a complete classification of automorphisms that act purely non-symplectically on K3 surfaces. In the talk I will give several examples by using elliptic fibrations. I will give also some results under the assumption that the action is just non-symplectic. This is a joint work with R. Bell, P. Comparin, J. Li, A. Rincon-Hidalgo, A. Zannardini.

4:30pm **Workshop & Math Colloquium: Klaus Hulek - SCGP 102**

**Speaker:** Workshop & Math Colloquium: Klaus Hulek

**Title:** Classification in algebraic geometry and the geometry of moduli spaces

**Abstract:** The classification of mathematical objects is an important problem in many branches of mathematics, notably also in algebraic geometry. Typically, these classification cannot be achieved by (finite) lists. Instead one builds a new algebraic variety whose points correspond to then isomorphism class of the objects to be classified — a moduli space. This leads to the questions: how does one construct such classifying spaces and what can one say about the geometry of these moduli spaces? I will discuss these question in exemplary cases (old and new).

### Friday, February 3rd

9:00am **Breakfast - SCGP Cafe**

**Title:** Breakfast

9:30am **Workshop Speaker: Daisuke Matsushita - SCGP 102**

**Speaker:** Daisuke Matsushita

**Title:** Strict nef divisor

**Abstract:** A line bundle  $L$  on a projective variety is said to be strict nef if  $L.C > 0$  for every effective curve  $C$  on  $X$ . We show that a strict nef divisor on IHS is ample.

10:30am **Coffee Break - SCGP Cafe**

**Title:** Coffee Break

11:00am **Workshop Speaker: Annalisa Grossi - SCGP 102**

**Speaker:** Annalisa Grossi

**Title:** Terminalizations of quotients of HK manifolds via symplectic actions.

**Abstract:** Terminalizations of symplectic quotients of HK manifolds are sources of new deformation classes of irreducible holomorphic symplectic varieties. In this talk I will describe all terminalizations of quotients by groups of automorphisms on Hilbert scheme of K3 surfaces and generalized Kummer varieties induced by symplectic automorphisms of the underlying K3 surface or abelian surface. I will sketch how to determine the second Betti number of these varieties and the fundamental group of the regular locus of these varieties. This is a joint work in progress with Bertini, Capasso, Debarre, Mauri and Mazzon.

12:00pm **Lunch - SCGP Cafe**

**Title:** Lunch